Does Competition Reduce the Risk of Bank Failure?

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A large theoretical literature shows that competition reduces banks' franchise values and induces them to take more risk. Recent research contradicts this result: When banks charge lower rates, their borrowers have an incentive to choose safer investments, so they will in turn be safer. However, this argument does not take into account the fact that lower rates also reduce the banks' revenues from performing loans. This paper shows that when this effect is taken into account, a U-shaped relationship between competition and the risk of bank failure generally obtains. (*JEL* G21, D43, E43)

The conventional wisdom of the banking literature holds that increased competition induces banks to take more risk.[1] By reducing a bank's franchise value, competition reduces the penalty for failure and thus the incentive for prudence. A key assumption in this literature is that banks invest in assets with exogenous distributions of returns. Recent work by Boyd and De Nicolo (2005), BDN henceforth, replaces this by the assumption that banks invest in loans. Following the seminal paper on credit rationing by Stiglitz and Weiss (1981), BDN assume that the risk of these loans is increasing in the loan interest rate. Hence a reduction in loan rates due to greater bank competition reduces the loans' probability of default. They also assume that loan defaults are perfectly correlated, in which case the loans' probability of default coincides with the bank's probability of failure. Hence they conclude that competition reduces the risk of bank failure.

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[1] See Keeley (1990), Besanko and Thakor (1993), Hellmann, Murdoch, and Stiglitz (2000), Matutes and Vives (2000), and Repullo (2004), among others.

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However, this finding does not necessarily hold in the (arguably more realistic) case of imperfect correlation of loan defaults, because then greater bank competition also reduces the interest payments from performing loans, which provide a buffer to cover loan losses. Thus, in addition to the *risk-shifting effect*, there is a *margin effect* that goes in the opposite direction, so the final effect on the risk of bank failure is in principle ambiguous.

Our basic setup follows that of BDN, except for the introduction of imperfect correlation in loan defaults. Specifically, we use a static model of Cournot competition in a market for entrepreneurial loans in which the probability of default of these loans is privately chosen by the entrepreneurs. The banks are funded with fully insured deposits and have no capital. To model imperfect default correlation, we use the single risk factor model of Vasicek (2002), according to which the default of an individual loan is driven by the realization of two risk factors: a systematic risk factor that is common to all loans, and an idiosyncratic risk factor. This model is very convenient, since it provides a closed form for the probability distribution of the default rate.[2] It also encompasses the cases of statistically independent defaults (when the weight of the systematic risk factor is zero) and perfectly correlated defaults (when the weight of the idiosyncratic risk factor is zero).

We show that the result in BDN is not robust to the introduction of even a small deviation from perfect correlation in loan defaults. Specifically, when the number of banks is sufficiently large, the risk-shifting effect is always dominated by the margin effect, so any additional entry would increase the risk of bank failure.

In less competitive loan markets the effect is ambiguous, so we resort to numerical solutions for a large range of parameterizations of the model. We show that in general there is a U-shaped relationship between competition (measured by the number of banks) and the risk of bank failure. In other words, in very concentrated markets the risk-shifting effect dominates, so entry reduces the probability of bank failure, whereas in very competitive markets the margin effect dominates, so further entry increases the probability of failure.

To check the robustness of these results, we first relax the assumption that deposits are fully insured. Without deposit insurance the deposit rate is no longer an exogenous parameter, but an endogenous variable that is derived from the depositors' participation constraint. We show that eliminating deposit insurance increases the risk of bank failure, because the higher cost of deposits translates into higher loan rates, which make loans riskier. But we still have a U-shaped relationship between competition and the risk of bank failure.

Next we consider a dynamic version of the model of Cournot competition in the loan market, in which banks that do not fail in one period have the

[2] The 99.9% quantile of this distribution is used for the computation of capital requirements in the framework proposed by the Basel Committee on Banking Supervision (2004), known as Basel II (Gordy 2003).

opportunity to lend to a new generation of entrepreneurs in the next period. This produces an endogenous franchise value that is lost upon failure, so banks have an incentive to be prudent. We show that the same U-shaped relationship between competition and the risk of bank failure obtains. We also show that the introduction of franchise values enhances bank stability (relative to the static setup), except in the case in which the number of banks is the one that minimizes the probability of failure, where it has no effect.

Finally, the same results (both static and dynamic) obtain when the Cournot model is replaced by a circular road model of competition in the loan market. The fact that the results are robust to the change of strategic variable from quantities (loan supplies) to prices (loan rates) suggests that they are likely to hold for a wide set of models of imperfect competition. Hence the conclusion is that when loan defaults are imperfectly correlated, the probability of bank failure is lowest in loan markets with moderate levels of competition, with higher probabilities of failure in either very competitive or very monopolistic markets.

To simplify the presentation, we abstract from competition in the deposit market by assuming that banks face a perfectly elastic supply of deposits at an interest rate (or expected return in the uninsured case) that is normalized to zero.[3] But our results for the Cournot model are robust to the introduction of an upward-sloping supply of deposits.[4] Indeed, the margin effect that we have identified is stronger in such a model, because greater bank competition not only reduces the interest payments from performing loans, but also increases the cost of deposit financing.

The rest of the paper is structured as follows. Section 1 presents the static model of Cournot competition with imperfectly correlated defaults. Section 2 characterizes the equilibrium of this model, and analyzes the effect of an increase in the number of banks on loan rates and probabilities of bank failure. Section 3 presents the numerical results on the U-shaped relationship between competition and the risk of bank failure. Section 4 shows that these results also obtain in a model without deposit insurance, in a dynamic version of the Cournot model with endogenous franchise values, and in a circular road model of loan rate competition. Section 5 discusses the implications of the results for the relationship between monetary and financial stability, and the welfare maximizing competition policy in banking. Section 6 contains our concluding remarks.

3 In models where banks invest in assets with exogenous distributions of returns, imperfect competition can only be introduced in the deposit market, but in models where banks face a downward-sloping demand for loans this is not needed.

4 Models where banks compete by simultaneously setting deposit and loan rates are more complicated to analyze, because the balance sheet identity (loans = deposits) is in general not satisfied. See, for example, the discussion in Yanelle (1997).

1. The Model

Consider an economy with two dates ($t=0,1$) and three classes of risk-neutral agents: entrepreneurs, banks, and depositors.

1.1 Entrepreneurs

There is a continuum of penniless entrepreneurs characterized by a continuous distribution of reservation utilities with support $\mathbb{R}_+$. Let $G(u)$ denote the measure of entrepreneurs that have reservation utility less than or equal to u.

Each entrepreneur i has a project that requires a unit investment at date 0, and yields a stochastic payoff at date 1:

$$R(p_i)=\begin{cases}1+\alpha(p_i), & \text{with probability } 1-p_i\\ 1-\lambda, & \text{with probability } p_i\end{cases}$$

where the probability of failure $p_i \in [0,1]$ is privately chosen by the entrepreneur at date 0. Following Allen and Gale (2000, Chapter 8), we assume that the success return of the project $\alpha(p_i)$ is positive and increasing in p_i. Thus riskier projects have a higher success return. In order to get interior solutions to the entrepreneur's choice of risk, we also assume that $\alpha(p_i)$ is concave in p_i and satisfies $\alpha(0)<\alpha'(0)$. The project's loss given failure λ is positive and smaller than 1, and to simplify the presentation we assume that it does not depend on p_i.

Project failures are correlated according to the *single risk factor* model of Vasicek (2002), in which the failure of the project of entrepreneur i is driven by the realization of a latent random variable:

$$y_i=-\Phi^{-1}(p_i)+\sqrt{\rho}\,z+\sqrt{1-\rho}\,\varepsilon_i, \tag{1}$$

where $\Phi(\cdot)$ denotes the cdf of a standard normal random variable and $\Phi^{-1}(\cdot)$ its inverse, z is a systematic risk factor that affects all projects, ε_i is an idiosyncratic risk factor that only affects the project of entrepreneur i, and $\rho\in[0,1]$ is a parameter that determines the extent of correlation in project failures. It is assumed that z and ε_i are standard normal random variables, independently distributed from each other as well as, in the case of ε_i, across projects.

The project of entrepreneur i fails when $y_i<0$. The deterministic term $-\Phi^{-1}(p_i)$ in (1) ensures that the probability of failure satisfies:

$$\Pr(y_i<0)=\Pr[\sqrt{\rho}\,z+\sqrt{1-\rho}\,\varepsilon_i<\Phi^{-1}(p_i)]=\Phi[\Phi^{-1}(p_i)]=p_i.$$

Notice that for $\rho=0$ the systematic risk factor does not play any role and we have statistically independent failures, while for $\rho=1$ the idiosyncratic risk factor does not play any role and we have perfectly correlated failures.

Entrepreneurs borrow from banks to fund their projects.[5] For any given loan rate r, entrepreneur i will choose p_i in order to maximize its expected payoff from undertaking the project, which is the success return net of the interest payment, $\alpha(p_i) - r$, times the probability of success, $1 - p_i$.[6] Let

$$u(r) = \max_{p_i}(1 - p_i)(\alpha(p_i) - r) \tag{2}$$

denote the maximum expected payoff that an entrepreneur can obtain when the loan rate is r. Since entrepreneurs only differ in their reservation utilities, the solution $p(r)$ to this problem and hence $u(r)$ do not depend on i. Moreover, for $\alpha(0) - \alpha'(0) < r < \alpha(1)$, the solution $p(r)$ will be interior[7] and characterized by the first-order condition:

$$(1 - p)\alpha'(p) - \alpha(p) + r = 0. \tag{3}$$

Hence, by the envelope theorem, we have $u'(r) = -(1 - p(r)) < 0$. Also, since $\alpha'(p) > 0$ and $\alpha''(p) \leq 0$, differentiating the first-order condition we get

$$p'(r) = \frac{1}{2\alpha'(p) - (1 - p)\alpha''(p)} > 0.$$

Thus the higher the loan rate, the higher the probability of failure chosen by the entrepreneurs. The positive effect of loan rates on entrepreneurs' optimal choice of risk will be denoted as the *risk-shifting effect*.[8]

Entrepreneur i will want to undertake a project when the loan rate is r if the reservation utility u_i is smaller than or equal to the expected payoff $u(r)$. Hence, the measure of entrepreneurs that want to borrow from the banks at the rate r is given by $G(u(r))$. Since each one requires a unit loan, the *loan demand function* is

$$L(r) = G(u(r)). \tag{4}$$

Clearly, for $0 \leq r < \alpha(1)$, we have $L(r) > 0$ and $L'(r) = G'(u(r))u'(r) < 0$. Let $r(L)$ denote the corresponding inverse loan demand function.

Consider now the continuum of entrepreneurs that want to undertake their projects when the loan rate is r. By our previous argument they all choose the same probability of failure $p = p(r)$. But then, by the law of large numbers, the *failure rate* x (the fraction of projects that fail) for a given realization

5 The role of banks could be rationalized by reference to some screening or monitoring services. To simplify the presentation, this will not be developed here.

6 With probability p_i the project fails, in which case by limited liability the entrepreneur gets a zero payoff, and the bank recovers $1 - \lambda$.

7 The corner $p = 0$ cannot be a solution if $\alpha'(0) - \alpha(0) + r > 0$, which gives $\alpha(0) - \alpha'(0) < r$, while the corner $p = 1$ cannot be a solution if $-\alpha(1) + r < 0$, which gives $r < \alpha(1)$.

8 Notice that $p'(r) > 0$ may also be derived from models without any risk-shifting, such as in Merton (1974). For example, if $\Gamma(R)$ were the cdf of the (continuous) return of the project, then $p(r) = \Pr(R < 1 + r) = \Gamma(1 + r)$, so $p'(r) = \Gamma'(1 + r) > 0$.

of the systematic risk factor z coincides with the probability of failure of a (representative) project i conditional on z; that is,

$$x=\gamma(z)=\Pr\left(-\Phi^{-1}(p)+\sqrt{\rho}\,z+\sqrt{1-\rho}\,\varepsilon_i<0\mid z\right)$$
$$=\Phi\left(\frac{\Phi^{-1}(p)-\sqrt{\rho}\,z}{\sqrt{1-\rho}}\right).$$

From here it follows that the cdf of the failure rate is $F(x)=\Pr(\gamma(z)\leq x)=\Pr\left(z\geq\gamma^{-1}(x)\right)$ (since $\gamma'(z)<0$), so using the definition of $\gamma(z)$ and the fact that $z\sim N(0,1)$, we get

$$F(x)=\Phi\left(\frac{\sqrt{1-\rho}\,\Phi^{-1}(x)-\Phi^{-1}(p)}{\sqrt{\rho}}\right). \tag{5}$$

For $\rho\in(0,1)$ the cdf $F(x)$ is continuous and increasing, with $\lim_{x\to0}F(x)=0$ and $\lim_{x\to1}F(x)=1$. It is also the case that $E(x)=\int_0^1 x\,dF(x)=p$.[9] Since $\partial F(x)/\partial p<0$, changes in the probability of failure p lead to a first-order stochastic dominance shift in the distribution of the failure rate x.

Moreover, it can be shown that $\partial F/\partial\rho\geq0$ if and only if $x\leq\Phi(\sqrt{1-\rho}\,\Phi^{-1}(p))$, which together with the fact that $E(x)=p$ imply that changes in the correlation parameter ρ lead to a mean-preserving spread in the distribution of the failure rate x. When $\rho\to0$ (independent failures) the distribution of the failure rate approaches the limit $F(x)=0$, for $x<p$, and $F(x)=1$, for $x\geq p$. The single mass point at $x=p$ implies that a fraction p of the projects fail with probability 1. And when $\rho\to1$ (perfectly correlated failures), the distribution of the failure rate approaches the limit $F(x)=\Phi(-\Phi^{-1}(p))=1-p$, for $0<x<1$. The mass point at $x=0$ implies that with probability $1-p$ no project fails, and the mass point at $x=1$ implies that with probability p all projects fail.

1.2 Banks

There are n identical banks that at date 0 are funded with fully insured deposits, have no capital,[10] and invest in a portfolio of entrepreneurial loans. The supply of deposits is perfectly elastic at an interest rate that is normalized to zero, and there are no intermediation costs. We assume that banks compete for loans à la Cournot, so the strategic variable of bank $j=1,...,n$ is its supply of loans l_j. The aggregate supply of loans $L=\sum_{j=1}^n l_j$ determines the loan rate $r(L)$, which in turn determines the probability of failure chosen by the entrepreneurs $p(r(L))$.

[9] To see this, let x_i be an indicator function that takes the value 1 when the project of the entrepreneur i fails (that is, when $y_i<0$), and the value 0 otherwise. Since $E(x_i)=\Pr(y_i<0)=p$, it must be the case that the expected value of the failure rate x also equals p.

[10] We briefly discuss the introduction of capital in Section 6.

The return of bank j's portfolio is stochastic: A random fraction x of its loans default, in which case the bank loses the interest r, as well as a fraction λ of the principal. Thus the bank gets $l_j(1+r)$ from the fraction $1-x$ of the loans that do not default, recovers $l_j(1-\lambda)$ from the fraction x of defaulted loans, and has to pay back l_j to the depositors, so by limited liability its payoff at date 1 is

$$\max\{l_j(1+r)(1-x)+l_j(1-\lambda)x-l_j,0\}=l_j\max\{r-(r+\lambda)x,0\}.$$

Hence bank j's *payoff function* is

$$\pi(l_j,l_{-j})=l_jh(L), \tag{6}$$

where l_{-j} denotes the vector of loan supplies of the other $n-1$ banks, and

$$h(L)=E\left[\max\{r(L)-(r(L)+\lambda)x,0\}\right] \tag{7}$$

is the banks' expected payoff per unit of loans. This expression may be rewritten as

$$h(L)=\int_0^{\widehat{x}(L)}\left[r(L)-(r(L)+\lambda)x\right]\,dF(x;\,p(r(L))), \tag{8}$$

where

$$\widehat{x}(L)=\frac{r(L)}{r(L)+\lambda} \tag{9}$$

is the bankruptcy default rate, and the distribution function of the default rate x is written so as to keep track of the effect of the (endogenous) probability of default of the loans $p(r(L))$. Thus, when choosing its supply of loans l_j, bank j takes into account the direct effect on the loan rate $r(L)$, as well as the indirect effect on the distribution of the default rate, $F(x;\,p(r(L)))$.

Note that $h(L)$ plays the role of the inverse demand function in a standard Cournot model. To guarantee the existence and uniqueness of equilibrium, we are going to assume that functional forms and parameter values are such that $h(L)$ is decreasing and concave.[11] This assumption is stronger than the assumption that the inverse loan demand function $r(L)$ is decreasing and concave. To see this, consider the extreme cases of $\rho=0$ (independent defaults) and $\rho=1$ (perfectly correlated defaults). When $\rho=0$, we have

$$h(L)=r(L)-(r(L)+\lambda)p(r(L)), \tag{10}$$

so $h'(L)<0$ if $p'(r)<(1-p(r))/(r(L)+\lambda)$. Assuming that $r(L)$ and $p(r)$ are linear (as we will do in Section 3 below), we also have $h''(L)<0$. On the other hand, when $\rho=1$, we have

$$h(L)=r(L)(1-p(r(L))), \tag{11}$$

[11] Obviously, we also need $h(0)>0$.

so $h'(L) < 0$ if $p'(r) < (1 - p(r))/r(L)$. And assuming that $r(L)$ and $p(r)$ are linear, we also have $h''(L) < 0$. Hence, in both cases (as well as in the intermediate cases with $0 < \rho < 1$) the risk-shifting effect $p'(r)$ should not be very large. The intuition is easy to explain: To get a downward sloping $h(L)$, the reduction in the loan rate $r(L)$ following an increase in L must not be compensated by a very large reduction in the probability of default $p(r(L))$. For low values of r and p, it would suffice to have $p'(r) < 1/\lambda$. This assumption is quite reasonable. For example, for $\lambda = 0.45$ (as we will assume in Section 3 below), it suffices that an increase of 1 percentage point in the loan rate does not lead to an increase of more than 2.2 percentage points in the probability of default.

2. Equilibrium

This section characterizes the (symmetric) Cournot equilibrium of our model of competition in the loan market, and analyzes the effect of an increase in the number of banks on equilibrium loan rates and equilibrium probabilities of bank failure.

The assumption that $h'(L) < 0$ and $h''(L) < 0$ implies that there is a unique symmetric equilibrium characterized by the first-order condition:[12]

$$\frac{L}{n}h'(L) + h(L) = 0. \tag{12}$$

From here it is immediate to derive the effects of competition on equilibrium aggregate lending and loan rates.

Proposition 1. An increase in the number of banks n increases equilibrium aggregate lending L and consequently reduces the equilibrium loan rate r.

Proof. Differentiating the first-order condition (12) and using the assumptions $h'(L) < 0$ and $h''(L) < 0$ gives

$$\frac{dL}{dn} = -\frac{h(L)}{Lh''(L) + (n+1)h'(L)} > 0.$$

But then $r'(L) < 0$ implies that

$$\frac{dr}{dn} = r'(L)\frac{dL}{dn} < 0.$$

■

Proposition 1 implies that the higher the competition among banks, the lower the probability of default of the loans in their portfolios.[13] However, this does not necessarily imply a reduction in their probability of failure.

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[12] The simple proof of existence and uniqueness of equilibrium may be found in Tirole (1998, p. 225).

[13] Boyd, De Nicolo, and Jalal (2006) and Berger, Klapper, and Turk-Ariss (2009) have documented this effect.

To see this observe that banks fail whenever the default rate x is greater than the bankruptcy default rate $\widehat{x}(L)$ defined in (9). Using the probability distribution of the default rate (5), the *probability of bank failure* is given by

$$q(L) = \Pr(x > \widehat{x}(L)) = \Phi\left(\frac{\Phi^{-1}(p(r(L))) - \sqrt{1-\rho}\,\Phi^{-1}(\widehat{x}(L))}{\sqrt{\rho}}\right). \quad (13)$$

Since equilibrium aggregate lending L is a function of the number of banks n, the effect of n on the probability of bank failure q is given by

$$\frac{dq}{dn} = q'(L)\frac{dL}{dn}.$$

But $dL/dn > 0$ by Proposition 1, which implies that higher competition leads to lower risk of bank failure if and only if the slope of the function $q(L)$ is negative.

Now, differentiating (13), we get

$$q'(L) = \frac{\Phi'(\cdot)}{\sqrt{\rho}}\left[\frac{d\Phi^{-1}(p(r(L)))}{dp}p'(r(L))r'(L) - \sqrt{1-\rho}\frac{d\Phi^{-1}(\widehat{x}(L))}{dx}\widehat{x}'(L)\right]. \quad (14)$$

Since $\Phi'(\cdot) > 0$ (it is a normal density), the sign of $q'(L)$ is the same as the sign of the term in square brackets, which has two components. The first one is negative, since $d\Phi^{-1}(p)/dp > 0$, $p'(r) > 0$, and $r'(L) < 0$, while the second one is positive (whenever $\rho < 1$), since $d\Phi^{-1}(x)/dx > 0$ and $r'(L) < 0$ implies that

$$\widehat{x}'(L) = \frac{\lambda r'(L)}{(r(L)+\lambda)^2} < 0.$$

The negative effect is the *risk-shifting effect* identified by BDN: More competition leads to lower loan rates, which in turn lead to lower probabilities of default, and hence safer banks. The positive effect is what may be called the *margin effect*: More competition leads to lower loan rates, and consequently lower revenues from performing loans, which provide a buffer against loan losses, so we have riskier banks. Depending on which of the two effects dominates, the impact of competition on the risk of bank failure may be positive or negative.

A few special cases are worth mentioning. When $\rho = 1$ (perfectly correlated defaults), the margin effect in (14) disappears, so we get the result in BDN: Competition always reduces the risk of bank failure.[14] When $p'(r) = 0$, the risk-shifting effect in (14) disappears, so competition always increases the risk

[14] Alternatively, substituting $\rho = 1$ in (13) gives $q(L) = p(r(L))$, which implies that $q'(L) = p'(r(L))r'(L) < 0$.

of bank failure. And when $\rho = 0$ (independent defaults), the default rate is deterministic (a fraction p of the loans default with probability 1), in which case for any number of banks the probability of bank failure is zero.[15] For $0 < \rho < 1$ and $p'(r) > 0$, the result is in general ambiguous. However, one can show that in very competitive loan markets, the risk-shifting effect is always dominated by the margin effect.

Proposition 2. For any correlation parameter $\rho \in (0, 1)$ and a sufficiently large number of banks n, the probability of bank failure q is increasing in n.

Proof. When n tends to infinity, the first-order condition (12) that characterizes the Cournot equilibrium becomes $h(L) = 0$, which by (8) implies that $\widehat{x}(L) = 0$, which in turn by (9) implies that $r(L) = 0$. Hence, in very competitive markets the loan rate approaches the deposit rate, which has been normalized to zero. Our assumption $\alpha(0) < \alpha'(0)$ implies that $\lim_{n\to\infty} p(r(L)) = p(0) > 0$, so we have

$$\lim_{n\to\infty} \frac{d\Phi^{-1}(p(r(L)))}{dp} = \frac{1}{\Phi'[\Phi^{-1}(p(0))]} < \infty$$

and

$$\lim_{n\to\infty} \frac{d\Phi^{-1}(\widehat{x}(L))}{dx} = \frac{1}{\Phi'[\Phi^{-1}(0)]} = \infty.$$

Hence by (14), we have $q'(L) > 0$ for sufficiently large n, which implies $dq/dn > 0$. ■

Proposition 2 shows that in loan markets with many banks, any additional entry increases the risk of bank failure. The intuition for this result is that as we get close to perfect competition, the margin between loan and deposit rates converges to zero. But since the probability of default of the loans is bounded away from zero (and banks have no capital), in the limit loan losses will always be greater than the intermediation margin, so banks will fail with probability 1. Hence the relationship between the number of banks and the probability of bank failure will be eventually increasing.

What happens in less competitive loan markets? To answer this question, in the next section we resort to numerical solutions for simple parameterizations of the model.

3. Numerical Results

In this section we compute the equilibrium of the model of competition in the loan market for a simple parameterization in which the inverse demand for

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15 With independent defaults ($\rho = 0$) banks never fail ($q = 0$), because the (deterministic) interest income per unit of loans, $(1 - p(r(L)))r(L)$, is greater than the (deterministic) loan losses per unit of loans, $p(r(L))\lambda$.

loans $r(L)$ and the entrepreneurial risk-shifting function $p(r)$ are linear, and examines how the probability of bank failure q changes with the number of banks n.

The critical parameters that determine the shape of the relationship between q and n are the correlation parameter ρ and the risk-shifting parameter $b = p'(r)$. By the results in Section 2 we know that q is decreasing in n when $\rho \to 1$ (the case of perfectly correlated defaults), and it is increasing in n when $b \to 0$ (the case of no entrepreneurial risk-shifting). Proposition 2 shows that q is increasing in n for sufficiently high n. Our numerical results shed light on what happens for $0 < \rho < 1$ and $b > 0$, and for smaller values of n.[16]

Specifically, we postulate an entrepreneurial risk-shifting function of the form

$$p(r) = a + br, \tag{15}$$

where $a > 0$ and $b > 0$, and an inverse demand for loans of the form

$$r(L) = c - dL, \tag{16}$$

where $c > 0$ and $d > 0$. The linear function $p(r)$ can be derived from a success return function of the form

$$\alpha(p) = \frac{1 - 2a + p}{2b}, \tag{17}$$

which implies the expected payoff function[17]

$$u(r) = \frac{(1 - a - br)^2}{2b}. \tag{18}$$

In this setup, parameter a is the minimum probability of default (that is, the one that would be chosen by the entrepreneurs for a zero loan rate), b is the entrepreneurial risk-shifting parameter, c is the maximum loan rate (for which the demand for loans reduces to zero), and d is the absolute value of the slope of the inverse loan demand function.

In our benchmark parameterization we take $a = 0.01$, $b = 0.5$, $c = 1$, and $d = 0.01$. This means that the demand for loans goes from 100 to 0 as loan rates go from 0% to 100%, and that the probability of default that corresponds to a loan rate of 0% is 1%, and to a loan rate of 2% is 2%. The loss given default parameter λ is set at 0.45, and the correlation parameter ρ is set at 0.2.[18] It should be noted that these parameters are chosen for the purpose of

16 The computations are carried out in Matlab. The program is available upon request.

17 Since $L(r) = G(u(r))$, one can show that (15) and (16) also imply $G(u) = (a + bc - 1 + \sqrt{2bu})/bd$.

18 The value of λ is the one specified in the Internal Ratings Based (Foundation Approach) of Basel II for senior claims on corporate, sovereign, and bank exposures not secured by recognized collateral. The value of ρ for these exposures ranges from 0.12 to 0.24. See Basel Committee on Banking Supervision (2004, par. 287 and 272).

illustrating the possible shapes of the relationship between the number of banks and the risk of bank failure. They are not intended to produce realistic values of variables, such as the loan rate r, the probability of loan default p, or the probability of bank failure q.

Figure 1 shows the relationship between the number of banks n (expressed in $\log_{10} n$, so n ranges from 1 to 10,000 banks) and the probability of bank failure q for three different values of the correlation parameter, $\rho = 0$, 0.2, and 1, with the other parameters at their benchmark levels. As noted in Section 2, with independent defaults ($\rho = 0$), banks never fail ($q = 0$). With perfectly correlated defaults ($\rho = 1$), the probability of bank failure is decreasing in the number of banks, which is the result in BDN. Interestingly, when $\rho = 0.2$, we have a U-shaped relationship between competition and the risk of bank failure, with a minimum q for $n = 3$.

Figure 2 shows the relationship between the number of banks n (expressed in $\log_{10} n$) and the probability of bank failure q for three different values of the entrepreneurial risk-shifting parameter, $b = 0$, 0.5, and 1, with the other parameters at their benchmark levels. As noted in Section 2, when $b = 0$, the risk-shifting effect disappears, so the margin effect makes the probability of bank failure increasing in the number of banks. For $b = 0.5$, we have the same U-shaped relationship already depicted in Figure 1. For higher values of the risk-shifting parameter such as $b = 1$, the risk-shifting effect becomes stronger, but the slope of the relationship eventually becomes positive (in this case from $n = 10$). In all these cases, as we get close to perfect competition the probability of bank failure converges to one. This is because when n tends to infinity, the margin between loan and deposit rates converges to zero, but with $\rho \in (0, 1)$ and $p(0) = a > 0$, loan losses are positive with probability 1.

It turns out that the U-shaped relationship between the number of banks n and the probability of bank failure q obtains for a very large set of parameter

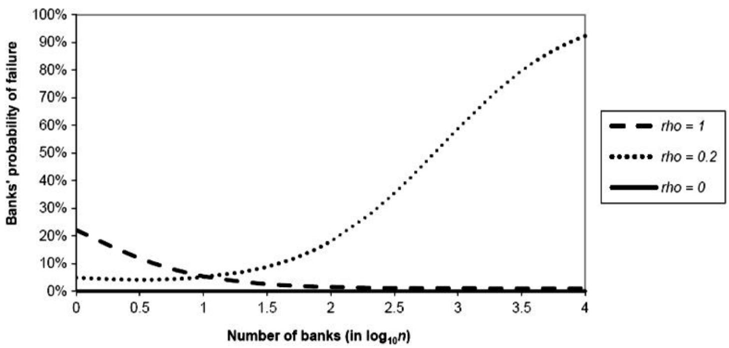

Figure 1
Competition and the risk of bank failure for different default correlations
This figure shows the relationship between the number of banks n and the probability of bank failure q in the Cournot model for different values of the correlation parameter ρ.

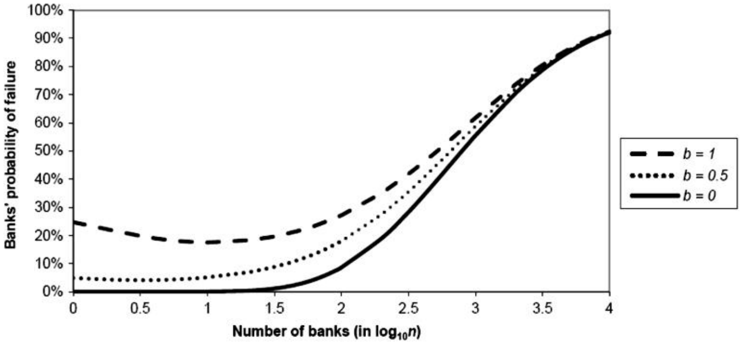

Figure 2
Competition and the risk of bank failure for different risk-shifting
This figure shows the relationship between the number of banks n and the probability of bank failure q in the Cournot model for different values of the risk-shifting parameter b.

values. Let $n_{\min}$ denote the number of banks that minimize q. Figure 3 illustrates the way in which the correlation parameter ρ and the entrepreneurial risk-shifting parameter b determine $n_{\min}$. Specifically, it shows the combinations of ρ and b for which $n_{\min} = 1, 2, 3, \ldots$. For low values of ρ or low values of b, we have $n_{\min} = 1$, so a monopolistic bank would minimize the probability of failure. Otherwise we have a U-shaped relationship: When the actual number of banks n is below (above) the corresponding $n_{\min}$, more (less) competition would reduce q. Higher correlation ρ and higher risk-shifting b increase $n_{\min}$, which reaches values greater than 100 when $\rho \to 1$, i.e., with perfectly correlated defaults.

It should be noted that similar results obtain for other functional forms for the inverse demand for loans $r(L)$, such as $r(L) = c - dL^{\delta}$ with $\delta > 1$ (to ensure concavity), and for the entrepreneurial risk-shifting function $p(r)$, such as $p(r) = a + br^{\eta}$ with $\eta > 0$. Thus, we conclude that too little and too much competition are generally associated with higher risks of bank failure.

Although our analysis has focused on the impact of changes in competition within the banking sector, the results could be easily extended to a situation in which the banking sector faces increased "outside" competition from financial markets. In particular, suppose that entrepreneurs have the option of funding their projects in a public debt market at an interest rate $\bar{r} > 0$. This outside option truncates the loan demand function at the rate $\bar{r}$. If the truncation is binding, the equilibrium loan rate would be $r = \bar{r}$, so an increase in competition coming from the financial markets would lead to a reduction in equilibrium loan rates. As before, the effect on the risk of bank failure would result from the combination of a negative risk-shifting effect and a positive margin effect, with the margin effect dominating for sufficiently small values of the market rate $\bar{r}$.

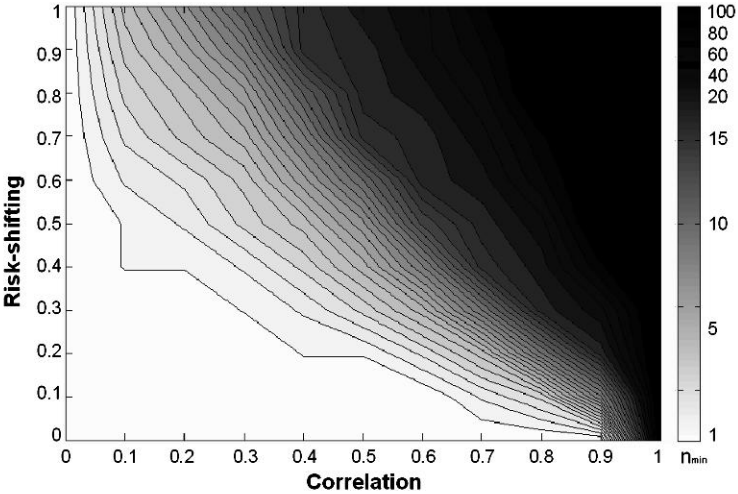

Figure 3
Number of banks that minimize the risk of bank failure in the Cournot model
This figure shows the number of banks $n_{\min}$ that minimize the probability of bank failure q in the Cournot model for different combinations of the correlation parameter ρ and the risk-shifting parameter b. For given values of ρ and b, the relationship between the number of banks n and the probability of bank failure q is decreasing until n reaches $n_{\min}$, and is increasing afterward. Darker areas correspond to higher values of $n_{\min}$.

4. Extensions

Three extensions of the model are analyzed in this section. First, we relax the assumption that deposits are fully insured. Second, we consider a dynamic version of the model of Cournot competition in the loan market in which banks that do not fail at any date t have the opportunity to lend to a new generation of entrepreneurs at date $t+1$. This generates an endogenous franchise value that is lost upon failure, so banks have an incentive to be prudent. Finally, we replace the Cournot model by a circular road model of competition in the loan market, in which loan rates are the banks' strategic variables. In all these extensions, our previous results on the relationship between competition and the risk of bank failure remain unchanged.

4.1 Uninsured deposits

Without deposit insurance, the deposit rate is no longer an exogenous parameter, but an endogenous variable that is derived from the depositors' participation constraint. Under risk-neutrality, such constraint requires that the expected return of deposits equals a risk-free rate that we normalize to zero.

Bank j's payoff function becomes

$$\pi(l_j, l_{-j}) = l_j E\left[\max\{r(L) - d - (r(L)+\lambda)x, 0\}\right],$$

where the deposit rate d satisfies the participation constraint

$$E\left[\min\{r(L) - (r(L) + \lambda)x, d\}\right] = 0.$$

But since

$$E\left[\max\{r(L) - d - (r(L) + \lambda)x, 0\}\right] + E\left[\min\{r(L) - (r(L) + \lambda)x, d\}\right]$$
$$= r(L) - (r(L) + \lambda)p(r(L)),$$

we can write

$$\pi(l_j, l_{-j}) = l_j h(L),$$

where

$$h(L) = r(L) - (r(L) + \lambda)p(r(L)).$$

Assuming, as before, that $h'(L) < 0$ and $h''(L) < 0$, the model without deposit insurance also has a unique symmetric equilibrium. Since the banks' expected payoff per unit of loans $h(L)$ coincides with (10), equilibrium loan rates are identical to those of the original model with $\rho = 0$. However, we have a positive probability of bank failure given by

$$q(L, d) = \Pr(x > \widehat{x}(L, d)) = \Phi\left(\frac{\Phi^{-1}(p(r(L))) - \sqrt{1-\rho}\,\Phi^{-1}(\widehat{x}(L, d))}{\sqrt{\rho}}\right),$$

where $\widehat{x}(L, d)$ is the new bankruptcy default rate defined by

$$\widehat{x}(L, d) = \frac{r(L) - d}{r(L) + \lambda}.$$

Figure 4 shows the relationship between the number of banks n and the probability of bank failure q for the cases of insured and uninsured deposits (with the benchmark functional forms and parameter values described in Section 3). Two results are worth mentioning. First, for any number of banks n, the probability of bank failure is lower in the model with deposit insurance than in the model without deposit insurance. In other words, deposit insurance increases the stability of the banking system. The reason for this is that in the absence of deposit insurance, depositors require higher deposit rates, which imply higher loan rates and hence riskier loans. Second, the model without deposit insurance features the same U-shaped relationship between the number of banks and the probability of bank failure.

4.2 A dynamic Cournot model

Consider a discrete time, infinite horizon model with n identical banks that at each date $t = 0, 1, 2, \ldots$ in which they are open raise fully insured deposits at

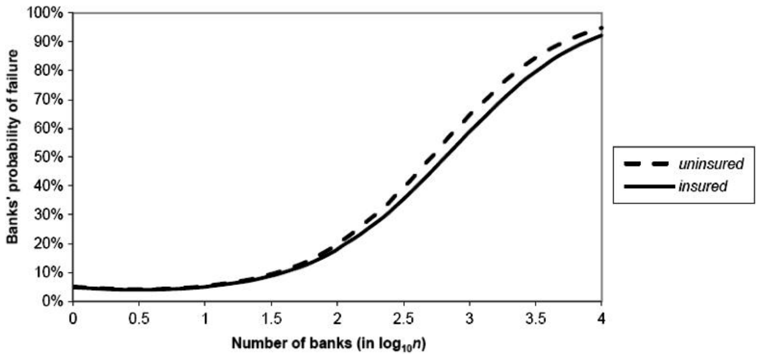

Figure 4
Competition and the risk of bank failure with insured and uninsured deposits
This figure shows the relationship between the number of banks n and the probability of bank failure q in the Cournot model with insured and uninsured deposits.

an interest rate that is normalized to zero and compete à la Cournot for loans to the continuum of entrepreneurs described in Section 1.1.

Assuming that banks are closed whenever the default rate x is greater than the bankruptcy default rate $\widehat{x}(L)$ defined in (9), the Bellman equation that characterizes the symmetric equilibrium of the dynamic model is

$$V_j = \max_{l_j} \beta \left[l_j h(L) + (1 - q(L)) V_j \right], \tag{19}$$

where $\beta < 1$ is the bank shareholders' discount factor, $h(L)$ is the banks' expected payoff per unit of loans given by (7), and $q(L)$ is the probability of bank failure given by (13). According to this expression, the franchise value of a bank that is open results from maximizing with respect to its supply of loans l_j (taking as given the equilibrium supplies of the other $n-1$ banks), an objective function that has two terms. The first one is the discounted expected payoff from current lending, $l_j h(L)$, and the second one is the discounted expected payoff of remaining open at the following date, which is the product of the probability of survival (one minus the probability of failure $q(L)$) and the franchise value V_j.[19]

It should be noted that with a single systematic risk factor, when one bank fails, all of them fail. Thus, there is no need to consider situations, like those in Perotti and Suarez (2002), where some banks may survive while others fail, so the surviving banks may increase their market power in the following periods. We do not discuss what happens after such failure, implicitly assuming that new banks enter the market. What is important for the analysis is that the failed banks lose their franchise value.

[19] See Fudenberg and Tirole (1991, Chapter 4) for a proof that one-stage deviations are sufficient to characterize subgame perfect equilibria.

Solving the Bellman equation (19) and setting $l_j = L/n$ gives the equilibrium aggregate lending L, as well as the banks' equilibrium franchise value V (the same for all j). As before, we assume that functional forms and parameter values are such that the dynamic model has a unique (noncollusive) equilibrium for all n. Then the relationship between the equilibrium of the static and the dynamic model is stated in the following result.

Proposition 3. Let L_s and L_d denote equilibrium aggregate lending in the static and the dynamic model, respectively, for a given number of banks n. Then, $q'(L_d) < 0$ implies $L_d > L_s$, $q'(L_d) > 0$ implies $L_d < L_s$, and $q'(L_d) = 0$ implies $L_d = L_s$.

Proof. The first-order condition (12) that characterizes the symmetric equilibrium of the static model is

$$L_s h'(L_s) + nh(L_s) = 0.$$

Differentiating the bank's objective function in (19), we get the first-order condition that characterizes the symmetric equilibrium of the dynamic model:

$$L_d h'(L_d) + nh(L_d) = nq'(L_d)V.$$

Hence, when $q'(L_d) < 0$, we have $L_d h'(L_d) + nh(L_d) < L_s h'(L_s) + nh(L_s)$. But since $h'(L) < 0$ and $h''(L) < 0$ imply that the function $Lh'(L) + nh(L)$ is decreasing, it follows that $L_d > L_s$. The second and third results are proved in the same manner. ■

The result in Proposition 3 shows that there is one case, namely when $q'(L_d) = 0$, in which aggregate lending, and consequently the probability of bank failure, are the same in both the static and the dynamic model. In all other cases we know the effect on aggregate lending, but to establish the effect on the probability of bank failure, we need to know the form of the function $q(L)$. Assuming that $q(L)$ is U-shaped, Proposition 3 implies that when the number of banks n is such that $q'(L_d) < 0$, we have $q(L_d) < q(L_s)$, and when the number of banks n is such that $q'(L_d) > 0$, we also have $q(L_d) < q(L_s)$. Hence, in both cases the probability of bank failure is smaller in the dynamic model than in the static model, so banks are generally safer in the model with endogenous franchise values. It is only in the case with $q'(L_d) = 0$ that both probabilities coincide.

We illustrate this result for our benchmark parameterization (for which the function $q(L)$ is indeed U-shaped) and $\beta = 0.96$. Figure 5 shows the relationship between the number of banks n and the probability of bank failure q in the static and in the dynamic model. In both cases the relationship is U-shaped, with the curve for the static model being everywhere above the curve for the dynamic model, except at the minimum in which they are tangent. The two curves are also tangent when n tends to infinity, because the equilibrium franchise value V tends to zero as we approach the perfect competition limit, in

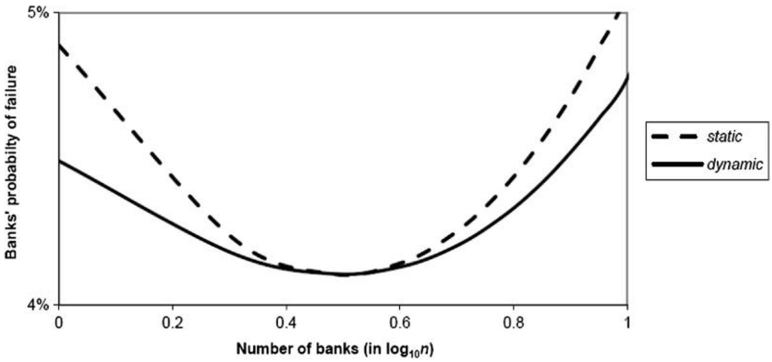

Figure 5
Competition and the risk of bank failure in the static and the dynamic model
This figure shows the relationship between the number of banks n and the probability of bank failure q in the static and the dynamic Cournot model.

which case the banks' objective function in the dynamic model coincides with the objective function in the static model.

The tangency result implies that Figure 3 also shows for the dynamic model the way in which the correlation parameter ρ and the entrepreneurial risk-shifting parameter b determine the effect of the number of banks on the probability of bank failure. Hence, we conclude that our results on the relationship between competition and the probability of bank failure are robust to the introduction of endogenous franchise values in our model of Cournot competition in the loan market.

It should be noted that we are focusing on the noncooperative equilibrium of the dynamic game, ignoring the possibility of collusive equilibria in which banks restrict their lending under the threat of reverting to the noncooperative equilibrium if a deviation occurs. In particular, n banks could sustain the monopoly outcome if

$$\max_{l_j} \beta \left[l_j h \left(l_j + (n-1)\frac{L_1}{n} \right) + \left[1 - q \left(l_j + (n-1)\frac{L_1}{n} \right) \right] V_n \right] \leq \frac{V_1}{n},$$

where V_n denotes the (noncooperative) franchise value of a bank when there are n banks in the market, and L_1 denotes the lending of a monopoly bank in the dynamic model. The left-hand side of this expression is the expected payoff that bank j could obtain by deviating from the collusive equilibrium, while the right-hand side is the expected payoff under collusion. It is well known that as n increases it is more difficult to sustain collusive equilibria. For example, for our numerical parameterization, this condition is satisfied for $n \leq 74$. Clearly, in this region the probability of bank failure would be $q(L_1)$, a constant independent of n.

4.3 A circular road model

We now examine the robustness of our results to changes in the nature of competition among banks. Specifically, we consider Salop's (1979) circular road model of price competition.[20] There are $n \geq 2$ banks located symmetrically on a circumference of unit length, and a continuum of measure 1 of entrepreneurs distributed uniformly on this circumference. We focus on the static version of the model, since the results for the dynamic version are similar to those obtained for the model of Cournot competition.

Entrepreneurs have the investment projects described in Section 1.1. They are ex-ante identical except for their location on the circumference, and have a zero reservation utility. To fund their projects they have to travel to a bank, which involves a transport cost μ per unit of distance.

To obtain the symmetric Nash equilibrium of the model of spatial competition, we first compute the demand for loans of bank j when it offers a loan rate r_j while the remaining $n-1$ banks offer the rate r.[21] Assuming that the transport cost μ is not too high, the market will be "totally covered," and bank j will have two effective competitors, namely banks $j-1$ and $j+1$. An entrepreneur located at distance θ from bank j and distance $1/n-\theta$ from bank $j+1$ will be indifferent between borrowing from j and borrowing from $j+1$ if the utility net of transport costs is the same; that is, if

$$u(r_j) - \mu\theta = u(r) - \mu\left(\frac{1}{n} - \theta\right).$$

Solving for θ in this equation yields

$$\theta(r_j, r) = \frac{1}{2n} + \frac{u(r_j) - u(r)}{2\mu}.$$

Taking into account the symmetric market area between bank j and bank $j-1$, and the fact that each entrepreneur requires a unit loan, we get the following demand for loans of bank j:

$$l(r_j, r) = \frac{1}{n} + \frac{u(r_j) - u(r)}{\mu}. \tag{20}$$

Notice that for $r_j = r$ (as will be the case in a symmetric equilibrium) we have $l(r, r) = 1/n$, so banks would get an equal share of the unit mass of borrowers.

[20] This model has been used in the context of banking by, among others, Chiappori, Perez-Castillo, and Verdier (1995) and Repullo (2004). In this model, the number of banks can be endogenized by introducing a fixed cost of entry. Since equilibrium profits are decreasing in the number of banks, an increase in competition would be equivalent to a reduction in the cost of entry.

[21] As in the original Salop (1979) model, we assume that banks do not price discriminate borrowers by their location.

Assuming that the supply of deposits is perfectly elastic at an interest rate that is normalized to zero, and following the same steps as in Section 1.2, bank j's payoff function may be written as

$$\pi(r_j, r) = l(r_j, r)h(r_j),$$

where

$$h(r_j) = \int_0^{\widehat{x}(r_j)} \left[r_j - (r_j + \lambda)x\right] \, dF(x; p(r_j)) \tag{21}$$

is bank j's expected payoff per unit of loans, and $\widehat{x}(r_j)$ is bank j's bankruptcy default rate defined by

$$\widehat{x}(r_j) = \frac{r_j}{r_j + \lambda}. \tag{22}$$

Assuming, as before, that functional forms and parameter values are such that there is a unique symmetric equilibrium, we can easily show that an increase in the number of banks reduces the equilibrium loan rate r. Since $p'(r) > 0$, this means that banks will have safer portfolios. However, as in the case of the model of Cournot competition, this does not imply a reduction in banks' probability of failure. To see this, observe that banks fail whenever the default rate x is greater than the bankruptcy default rate $\widehat{x}(r)$ defined in (22). Using the probability distribution of the default rate defined in (5), the probability of bank failure is given by

$$q(r) = \Pr\left(x > \widehat{x}(r)\right) = \Phi\left(\frac{\Phi^{-1}(p(r)) - \sqrt{1-\rho}\,\Phi^{-1}(\widehat{x}(r))}{\sqrt{\rho}}\right). \tag{23}$$

Since the equilibrium loan rate r is a function of the number of banks n, the effect of n on the probability of bank failure q is given by

$$\frac{dq}{dn} = q'(r)\frac{dr}{dn}.$$

But $dr/dn < 0$, which implies that higher competition leads to lower risk of bank failure if and only if the slope of the function $q(r)$ is positive.

Now, differentiating (23), we get

$$q'(r) = \frac{\Phi'(\cdot)}{\sqrt{\rho}}\left[\frac{d\Phi^{-1}(p(r))}{dp}p'(r) - \sqrt{1-\rho}\,\frac{d\Phi^{-1}(\widehat{x}(r))}{dx}\widehat{x}'(r)\right]. \tag{24}$$

Since $\Phi'(\cdot) > 0$ (it is a normal density), the sign of $q'(r)$ is the same as the sign of the term in square brackets, which has two components. The first one is positive, since $d\Phi^{-1}(p)/dp > 0$ and $p'(r) > 0$, while the second one is negative (whenever $\rho < 1$), since $d\Phi^{-1}(x)/dx > 0$ and $\widehat{x}'(r) > 0$. As in the model of Cournot competition, the first component captures the risk-shifting effect, while the second component captures the margin effect. Depending on which of the two effects dominates, the impact of competition on the risk of bank failure may be positive or negative. But, as in Proposition 2, one can show that when loan defaults are imperfectly correlated, the margin effect dominates in very competitive markets.

To illustrate what happens in less competitive markets, we resort to numerical solutions. As in Section 3, we postulate the linear risk-shifting function $p(r) = a + br$, for which the expected payoff function $u(r)$ is given by (18). For our benchmark parameterization, we take the minimum probability of default $a = 0.01$, the loss given default parameter $\lambda = 0.45$, and the transport cost parameter $\mu = 1$.

As in the case of the model of Cournot competition, we get a U-shaped relationship between the number of banks n and the probability of bank failure q. If we let $n_{\min}$ denote the number of banks that minimize the probability of failure, Figure 6 illustrates the way in which the correlation parameter ρ and the entrepreneurial risk-shifting parameter b determine $n_{\min}$. For low values of ρ or low values of b, we have $n_{\min} = 2$, so a duopoly would minimize the probability of failure. Higher correlation ρ and higher risk-shifting b increase $n_{\min}$, which reaches values greater than 100 when $\rho \to 1$, i.e., with perfectly correlated defaults.

Hence, we conclude that our results remain unchanged when we replace the Cournot model by a circular road model of competition in the loan market.

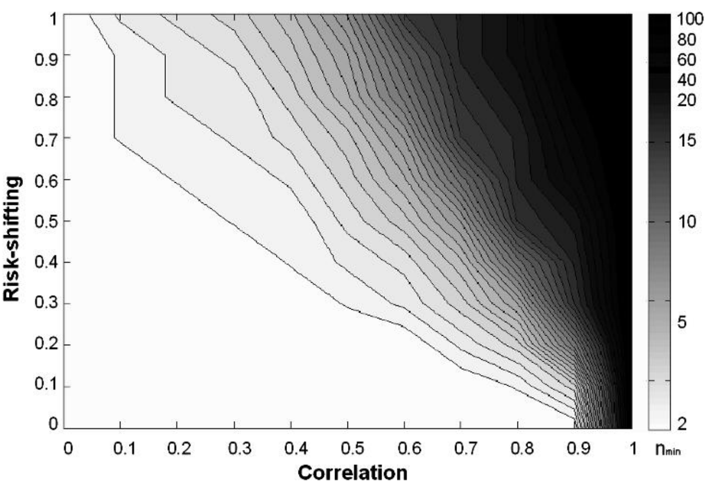

Figure 6
Number of banks that minimize the risk of bank failure in the circular road model
This figure shows the number of banks $n_{\min}$ that minimize the probability of bank failure q in the circular road model for different combinations of the correlation parameter ρ and the risk-shifting parameter b. For given values of ρ and b, the relation between the number of banks n and the probability of bank failure q is decreasing until n reaches $n_{\min}$, and is increasing afterward. Darker areas correspond to higher values of $n_{\min}$.

The fact that the results are robust to the change of strategic variable from quantities (loan supplies) to prices (loan rates) suggests that they are likely to hold for a wide set of models of imperfect competition. The general conclusion is that when loan defaults are imperfectly correlated, the probability of bank failure is lowest in loan markets with moderate levels of competition, with higher probabilities of failure in either very competitive or very monopolistic markets.

5. Discussion

This section discusses the effect of changes in the deposit rate in order to assess the relationship between monetary and financial stability, and the implications of our results for the welfare maximizing competition policy in banking. To simplify the presentation, the discussion will be conducted in the context of the static model of Cournot competition presented in Section 1.

5.1 Monetary and financial stability

Our model may be used to discuss the relationship between monetary and financial stability, in particular the effect of a tightening of monetary policy on the risk of bank failure. For this we replace the deposit rate by a risk-free rate i, which proxies the policy rate set by the central bank. Bank j's payoff function now becomes

$$\pi(l_j, l_{-j}) = l_j E\left[\max\{r(L) - i - (r(L) + \lambda)x, 0\}\right].$$

The comparative static analysis of the equilibrium effects of an increase in the risk-free rate i is not straightforward, so we illustrate them numerically. Figure 7 shows the relationship between the number of banks n (expressed

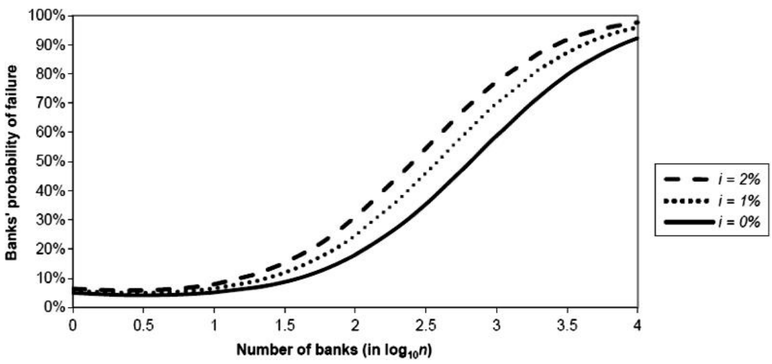

Figure 7
Competition and the risk of bank failure for different risk-free rates
This figure shows the relationship between the number of banks n and the probability of bank failure q in the Cournot model for different values of the risk-free rate i.

in $\log_{10} n$) and the probability of bank failure q for three different values of the risk-free rate, $i = 0$, 0.01, and 0.02, with the other parameters at their benchmark levels. In all cases the relationship is U-shaped, with higher values of the risk-free rate associated with higher values of the probability of bank failure. The fact that a tightening of monetary policy leads to an increase in the probability of bank failure is explained by a combination of a positive risk-shifting effect (that follows from the increase in the loan rate r) and a positive margin effect (that follows from the reduction in the intermediation margin $r - i$). Hence, in this case the margin effect reinforces the risk-shifting effect.

Thus, the model provides a framework for understanding the historical evidence of cases where the concern of a central bank for the solvency of the banking system was a major factor in an excessively expansionary monetary policy (see Goodhart and Schoenmaker 1995 and the references therein).

5.2 Welfare analysis

In our risk-neutral economy, social welfare may be evaluated by simply adding the expected payoffs of entrepreneurs, bank shareholders, depositors, and the government (as deposit insurer). In addition, we are going to assume that the failure of the banking system entails a social cost $C > 0$,[22] which captures the administrative costs of liquidating banks and paying back depositors, as well as the negative externalities associated with such failure (breakup of lending relationships, distortion of the payment system, etc.).

Since depositors are fully insured, they get a return that just covers the opportunity cost of their funds, so their net payoff is zero. The net expected payoff of an entrepreneur i that undertakes her project at the rate r is $(1 - p(r))(\alpha(p(r) - r) - u_i$, where u_i denotes her reservation utility. Hence, the net expected payoff of the entrepreneurs that undertake their projects at the rate r is

$$L(r)(1 - p(r))(\alpha(p(r)) - r) - \int_0^{u(r)} u \, dG(u).$$

The expected payoff of bank shareholders is

$$L(r) \int_0^{\widehat{x}(r)} [r - (r + \lambda)x] \; dF(x; p(r)).$$

Finally, the expected payoff of the government is

$$L(r) \int_{\widehat{x}(r)}^1 [r - (r + \lambda)x] \; dF(x; p(r)) - q(r)C,$$

where the first term is the expected liability of the deposit insurer (the expected value of the bank losses that obtain when the default rate x is greater

[22] Recall that since there is a single factor of systematic risk, if one bank fails all of them fail.

than the bankruptcy default rate $\widehat{x}(r)$), and the second term is the product of the probability of bank failure $q(r)$ by the social cost of failure C. Adding up the previous expressions, taking into account that the expected value of the default rate is the probability of default $p(r)$, and making use of the fact that $r = r(L)$, we obtain the following social welfare function:

$$\begin{aligned} W(L) = L\,[(1 - p(r(L)))\alpha(p(r(L))) - \lambda p(r(L))] \\ - \int_0^{u(r(L))} u\, dG(u) - q(L)C. \end{aligned} \tag{25}$$

The first term in this expression is the expected return of the projects that are undertaken, the second term is the opportunity cost of the entrepreneurs that undertake them, and the third term is the expected social cost of bank failure.

Differentiating (25) with respect to aggregate lending L, and making use of the definition of $u(r)$ in (2), the definition of $L(r)$ in (4), and the fact that $G(u(r(L))) = L$ implies $G'(u(r(L)))u'(r(L))r'(L) = 1$, we get

$$\begin{aligned} W'(L) = L\left[(1 - p(r(L)))\alpha'(p(r(L))) - \alpha(p(r(L))) - \lambda\right] p'(r(L)r'(L) \\ + [(1 - p(r(L)))r(L) - \lambda p(r(L))] - q'(L)C. \end{aligned}$$

The first term in this expression is positive, since $p'(r) > 0$, $r'(L) < 0$, and the function $(1 - p)\alpha(p) - \lambda p$ is concave, with a slope for $p = p(0)$ equal to $-\lambda < 0$.[23] The second term is the expected payoff of a bank loan, which should be positive except for large values of n for which it approaches the limit $-\lambda p(0) < 0$. Finally, we have seen that $q(L)$ is generally U-shaped, so the third term is positive (negative) for low (high) values of L.

Since by Proposition 1 aggregate lending L is an increasing function of the number of banks n, we conclude that the number of banks that maximizes social welfare, denoted n^*, is in general greater than the number $n_{\min}$ that minimizes the probability of bank failure, and satisfies $dn^*/dC < 0$. Moreover, as the social cost of bank failure C goes up, the optimal number of banks n^* approaches $n_{\min}$. Hence, we conclude that if bank failures generate some negative externalities, the welfare maximizing competition policy in banking will be characterized by entry restrictions that leave banks some monopoly rents in order to reduce their risk of failure.

6. Concluding Remarks

This article has investigated the effects of increased competition on the risk of bank failure in the context of a model in which (i) banks invest in entrepreneurial loans; (ii) the probability of default of these loans is endogenously

[23] To see this, use the fact that for $r = 0$ the first-order condition (3) that characterizes the entrepreneurs' choice of p becomes $(1 - p)\alpha'(p) - \alpha(p) = 0$.

chosen by the entrepreneurs; and (iii) loan defaults are imperfectly correlated. We show that there are two opposite effects. The *risk-shifting effect* identified by Boyd and De Nicolo (2005) follows from (i) and (ii) and works as follows: More competition leads to lower loan rates, which in turn lead to lower probabilities of loan default, and hence safer banks. The *margin effect* follows from (iii) and works as follows: More competition leads to lower loan rates, and consequently lower revenues from performing loans, which provide a buffer against loan losses, so we have riskier banks. The results show that the risk-shifting effect tends to dominate in monopolistic markets, whereas the margin effect dominates in competitive markets, so a U-shaped relationship between competition and the risk of bank failure generally obtains.

Our analysis has focused on a moral hazard model of the credit market, but similar results could be derived for an adverse selection model à la Stiglitz and Weiss (1981), where the average probability of default of a loan portfolio is increasing in the loan rate.

Finally, it should be noted that although we have assumed that banks have no capital, allowing bank shareholders to contribute costly capital would not change the results. To see this, suppose that in the static Cournot model bank j funds its lending using a proportion k_j of capital and a proportion $1-k_j$ of deposits. Then the bank's payoff at date 1 when a fraction x of its loans default would be

$$l_j\left[\max\{(1+r)(1-x)+(1-\lambda)x-(1-k_j),0\}-(1+\delta)k_j\right]$$
$$=l_j\left[\max\{r+k_j-(r+\lambda)x,0\}-(1+\delta)k_j\right],$$

where $\delta > 0$ denotes the cost of bank capital. From here it follows that bank j's payoff function would be

$$\pi(l_j,k_j,l_{-j})=l_j\left[\int_0^{\widehat{x}(L,k_j)}\left[r(L)+k_j-(r(L)+\lambda)x\right]\right.$$
$$\left.\times\, dF(x;\,p(r(L)))-(1+\delta)k_j\right],$$

where

$$\widehat{x}(L,k_j)=\frac{r(L)+k_j}{r(L)+\lambda}$$

is the default rate for which bank j fails. Differentiating the bank's payoff function with respect to k_j gives

$$\frac{\partial\pi(l_j,k_j,l_{-j})}{\partial k_j}=l_j\left[F(\widehat{x}(L,k_j);\,p(r(L)))-(1+\delta)\right]<0,$$

since F is a cdf. Hence the solution will always be at the corner $k_j = 0$, so banks will not hold any capital.[24]

This result would not necessarily extend to a dynamic setup, because for sufficiently high franchise values bank shareholders may want to contribute some capital in order to reduce the probability of losing the franchise.[25] Exploring this in detail, however, is beyond the scope of this article.

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24 Of course, banks may hold capital because of the existence of a capital requirement. See Martinez-Miera (2009) for an analysis of the effects of bank capital regulation in a model with a risk-shifting effect.

25 See Elizalde and Repullo (2007) for an analysis of capital buffers in a dynamic model of a bank whose credit risk is driven by a single risk factor à la Vasicek.

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